THE ROBUST COMPONENT STRUCTURE OF DENSE REGULAR GRAPHS

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Consider the classical result of Dirac that every graph on $n \geq 3$ vertices with minimum degree at least $n/2$ contains a Hamilton cycle. Suppose we wish to strengthen this by reducing the degree threshold at the expense of introducing some additional condition(s).

Bollobás and Häggkvist independently conjectured that any t-connected regular graph on n vertices with degree at least $n/(t+1)$ contains a Hamilton cycle.

This had already been proved for $t = 2$ by Jackson. A counterexample due to Jung and independently Jackson, Li and Zhu showed the conjecture to be false for $t \geq 4$.

We asymptotically prove the final case of this conjecture. We show that whenever $\varepsilon > 0$, every sufficiently large 3-connected regular graph on n vertices with degree at least $n/4 + \varepsilon n$ contains a Hamilton cycle.

This result is just one application of a general structural result for dense regular graphs, and we will sketch how to use it in this case. Our methods involve the notion of robust expansion, which has recently been used to solve several longstanding problems, notably Kelly's conjecture on Hamilton decompositions of regular tournaments and the 1-factorisation conjecture.

Roughly speaking, a graph is robustly expanding if it still expands after the deletion of a small fraction of its vertices and edges. Our main result states that every dense regular graph can be partitioned into 'robust components', each of which is a robust expander or a bipartite robust expander.

Open problems which may be susceptible to our tools are also discussed.

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