The chromatic numbers of random subgraphs of distance graphs

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Our talk is concerned with the classical Nelson–Hadwiger problem on finding the chromatic numbers of distance graphs in $\mathbb{R}^n$. We mainly consider a class of graphs $G(n,r,s) = (V(n,r), E(n,r,s))$ defined as follows:

$$V(n,r) = \{\mathbf{x} = (x_1, \ldots, x_n) : \ x_i \in \{0,1\}, \ x_1 + \ldots + x_n = r\}, \quad E(n,r,s) = \{\{\mathbf{x}, \mathbf{y}\} : \ (\mathbf{x}, \mathbf{y}) = s\},$$

where $(\mathbf{x}, \mathbf{y})$ is the Euclidean scalar product. In particular, recently the chromatic number of $G(n,3,1)$ was found by J. Balog, A. Kostochka, A. Raigorodskii (see [1]).

We study the random graphs $\mathcal{G}(G(n,r,s),p)$ whose edges are chosen independently from the set $E(n,r,s)$ each with probability p. We find concentration results for the independence numbers of such graphs and bounds for their chromatic numbers. We also study some algorithmic aspects of the above-mentioned questions.

References

[1] J. Balogh, A.V. Kostochka, A.M. Raigorodskii, *Coloring some finite sets in* $\mathbb{R}^n$, Discussiones Mathematicae Graph Theory, 33 (2013), N1, 25 - 31.

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