Stein Couplings to Show Limit Laws for Fringe Trees

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Abstract

The binary search tree or Quicksort is the most used of all sorting algorithm, since it is both fast and simple. The random recursive tree is another extensively studied random tree. We have examined fringe trees in these two types of random trees, since the study of such subtrees appears to be an effective approach towards defining characteristics also of the whole tree. By using a representation of Devroye for the binary search tree, and a wellknown bijection between recursive trees to binary trees, we use different applications of Stein's method to show both new general results on the asymptotic distributions for random variables on fringe trees, as well as provide more direct proofs of several earlier results in the field. The use of Stein couplings leads to simple proofs that the number of fringe trees in the binary search tree and the random recursive tree of size k, $k < n$ where $k \to \infty$ converges to a Poisson distribution. Furthermore, combining these results and another version of Stein's method, we can also show that for $k = o(\sqrt{n})$ the number of fringe trees in both types of random trees converges to a normal distribution. The general results on fringe trees also lead to simple proofs of a broad range of problems relating to random trees; one such example is the asymptotic distribution of the number of protected nodes in the binary search tree.