

Title: Sperner's Theorem and a Problem of Erdős-Katona-Kleitman.

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Abstract: A central result in extremal set theory is the celebrated theorem of Sperner from 1928, which gives the size of the largest family of subsets of $[n]$ not containing a 2-chain $F_1 \subsetneq F_2$. Erdős extended this theorem to determine the largest family without a k -chain $F_1 \subsetneq F_2 \subsetneq \dots \subsetneq F_k$. Erdős and Katona, followed by Kleitman, asked how many chains must appear in families with sizes larger than the corresponding extremal bounds.

In 1966, Kleitman resolved this question for 2-chains, showing that the number of such chains is minimized by taking sets as close to the middle level as possible. Moreover, he conjectured the extremal families were the same for k -chains, for all k . In this talk we present a first progress on this problem, verifying Kleitman's conjecture for the families whose size is at most the size of the $k + 1$ middle levels. We also characterize all extremal configurations.

Joint work with Shagnik Das and Benny Sudakov.